PHYSICAL REVIEW B **81**, 100412(R) (2010)

Kondo cloud mediated long-range entanglement after local quench in a spin chain

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(Received 23 February 2010; published 24 March 2010)

We show that, in the gapless Kondo regime, a single local quench at one end of a Kondo spin chain induces a fast and long-lived oscillatory dynamics. This quickly establishes a high-quality entanglement between the spins at the opposite ends of the chain. This entanglement is mediated by the Kondo cloud, attains a constant high value independent of the length for large chains, and shows thermal robustness. In contrast, when the Kondo cloud is absent, e.g., in the gapped dimer regime, only finite-size end to end effects can create some entanglement on a much longer time scale for rather short chains. By decoupling one end of the chain during the dynamics, one can distinguish between this end-end effect which vanishes, and the global Kondo cloud mediated entanglement, which persists. This quench approach paves the way to detect the elusive Kondo cloud through the entanglement between two individual spins. Our results show that nonperturbative cooperative phenomena from condensed matter may be exploited for quantum information.

DOI: 10.1103/PhysRevB.81.100412 PACS number(s): 75.10.Pq, 03.65.Ud, 03.67.Hk

I. INTRODUCTION

Entanglement in many-body spin systems is a topic of high interest[1–7] where "quantum correlations" or long-range entanglement between individual spins is notoriously difficult to achieve. Here we propose to generate such entanglement through a nonperturbative dynamical mechanism which requires only a *minimal action* on a spin chain, namely, a sudden quench of a single bond. We consider spin chains with a magnetic impurity[8] located at one end; under certain conditions, the ground state of the chain is such that the impurity is maximally entangled to the block of spins forming the Kondo cloud (KC).[6,7,9] We will show that, during the dynamics induced by the quench, it is the Kondo cloud which mediates entanglement between the two ending spins of the chain. The generated entanglement is measurable through two spin correlations and enables a direct detection of the presence of a KC within the system before quenching. This is important since, despite several manifestations, the direct observation of a KC is still an open challenge.[10] Besides being a quantum informational tool for probing a condensed-matter paradigm, the long-range entanglement created by local quenching is *substantial* and potentially useful for linking separated quantum registers.[11]

Though entanglement in condensed-matter systems is typically very short ranged,[2] there have been a few other proposals for long-distance entanglement—however, they come at a high price. For example, there are proposals exploiting weak couplings of two distant spins to a spin chain,[12,13] which have limited thermal stability or very long time scale of entanglement generation. Otherwise, a dynamics has to be induced by large-scale changes to the Hamiltonian of a system,[14] as opposed to the minimal change required in our proposal. Additionally, the exploitation of a nonperturbative cooperative feature of a condensed-matter system, such as a Kondo cloud, has not been addressed so far.

Quantum spin chains allow to investigate nonperturbative phenomena arising from the presence of impurities.[15] For instance, the spin-chain Kondo model[6,7,16] arises when a magnetic impurity[8] is coupled to the end of an antiferromagnetic J_1-J_2 spin-1/2 chain, where $J_1(J_2)$ is the (next) nearest-neighbor coupling. The model supports a crossover from a gapless Kondo regime (KR) for $J_2<J_2^c=0.2412J_1$ to a gapped dimerized regime (DR) for $J_2>J_2^c$. It is called a Kondo model since the impurity spin forms, in the KR, an effective singlet with a the spins inside the KC. Recently, the entanglement of the ground state of this system has been characterized by varied means.[6,7] The Kondo spin chain is described by the Hamiltonian,

$$H_I = J'(J_1\sigma_1\cdot\sigma_2 + J_2\sigma_1\cdot\sigma_3) + J_1\sum_{i=2}^{N-1}\sigma_i\cdot\sigma_{i+1} + J_2\sum_{i=2}^{N-2}\sigma_i\cdot\sigma_{i+2}, \quad (1)$$

where $\sigma_i=(\sigma_i^x,\sigma_i^y,\sigma_i^z)$ is the vector of Pauli operators at site i, $J_1>0(J_2>0)$ is the antiferromagnetic (next) nearest-neighbor coupling constants (we always put $J_1=1$), $J'<1$ is the impurity bond strength, and N is the number of spins, which we assume to be even through this Rapid Communication. H_I is the initial Hamiltonian of our dynamical scheme. Tuning J_2 enables one to shift the spin chain between the Kondo and DRs.

II. LONG-RANGE DISTANCE-INDEPENDENT ENTANGLEMENT IN THE KONDO REGIME

We consider the *finite* Kondo chain [Eq. (1)] in its ground state $|GS_I\rangle$. We, then, pertinently quench the coupling at the opposite end of the impurity allowing for the dynamics to develop entanglement. We show that, in the KR, the entanglement between the two ending spins oscillates between high peaks with a periodicity determined by J' while the dynamics is very fast (thereby decoherence hardly gets time to act) and is robust against thermal fluctuations. In the dimer phase, the dynamics is much slower, qualitatively different and, in finite chains, it generates some entanglement due to unavoidable end to end effects, which are drastically tamed if one "cuts off" the impurity from the chain during the dynamics. In the KR, cutting off the impurity has minimal effect on the final entanglement between the ending spins since, here, the process is mediated by the cloud.

Entanglement is expected to be very different[6,7,9] as J_2 crosses J_2^c since, for $J_2 < J_2^c$, there is a characteristic length—the so-called Kondo screening length ξ—determined only by J' through $\xi \propto e^{\alpha/\sqrt{J'}}$, where α is a constant. In the KR, for any given J' and N, ξ determines the size of a domain whose spins are maximally entangled with the impurity spin sitting at the origin.[9]

Initially, the system is assumed to be in the ground state $|GS_I\rangle$ of H_I. A minimal quench modifies only the couplings of the Nth spin by the amount J' [same as J' in Eq. (1)] so that H_I is changed to

$$H_F = J'(J_1\sigma_1 \cdot \sigma_2 + J_2\sigma_1 \cdot \sigma_3 + J_1\sigma_{N-1} \cdot \sigma_N + J_2\sigma_{N-2} \cdot \sigma_N) + J_1 \sum_{i=2}^{N-2} \sigma_i \cdot \sigma_{i+1} + J_2 \sum_{i=2}^{N-3} \sigma_i \cdot \sigma_{i+2}. \quad (2)$$

Since $|GS_I\rangle$ is not an eigenstate of H_F it will evolve as $|\psi(t)\rangle = e^{-iH_F t}|GS_I\rangle$. An entanglement $E(N,t,J')$ between the ending spins emerges as a result of the above evolution. To compute $E(N,t,J')$, we first obtain the reduced density matrix $\rho_{1N}(t) = \mathrm{tr}_{\widehat{1N}}|\psi(t)\rangle\langle\psi(t)|$ of spins 1 and N by tracing out the remaining spins from the state $|\psi(t)\rangle$. Then, we evaluate $E(N,t,J')$ in terms of a measure of entanglement valid for arbitrary mixed states of two qubits called the concurrence.[17] Entanglement takes its maximum E_m at an optimal time t_{opt} and an optimal coupling J'_{opt} such that $E_m = E(N,t_{opt},J'_{opt})$. As we shall see, J'_{opt} is not a perturbation of J_1 and J_2. If, as expected from scaling in the KR,[9,18] the dependence on N and t can be accounted for by a redefinition of J' (equivalently ξ), then t_{opt} and J'_{opt} cannot be independent quantities. Our numerical analysis shows indeed that, in the KR, $t_{opt} \propto N$ and that J'_{opt} yields $\xi = N-2$ since $\xi \propto e^{\alpha/\sqrt{J'}}$ one gets $t_{opt} \propto N \propto e^{\alpha/\sqrt{J'_{opt}}}$.

For our choice of J', J_1, and J_2, the spin-chain dynamics is not analytically solvable and one has to resort to numerical simulations. Recent methods of many-body simulations allow handling exponentially big Hilbert spaces with pertinent truncations. Here, for $N > 20$, we use the time-step targeting method, based on the density matrix renormalization group (DMRG) algorithm.[19] For $N < 20$, we resort to exact diagonalization.

We focus only on the first period of the entanglement evolution in both regimes since both decoherence and numerical errors make it unwise to wait for longer times. Figure 1(a) shows that fast long-lived (nondecaying) periodic oscillations with a period of $2t_{opt}$ characterize the time evolution in the KR and that the maximal entanglement is achieved when the impurity coupling J' equals the value J'_{opt} associated to a KC of size $\xi = N-2$ (the KC generated by the impurity sitting on the left side touches the other side of the chain); in the DR, the dynamics appears more dispersive and not oscillatory for any J'. In Fig. 1(b), we plot the maximum of entanglement, E_m, induced by bond quenching as a function of the length N: though the entanglement decreases as N increases, its value, in the KR, stays rather high and becomes almost *distance independent* for very long chains; furthermore, as N increases, the entanglement generated in the KR is significantly bigger than the one in the DR. Despite its lower value, achieving entanglement in the DR costs more time, as shown in Fig. 1(c). It is also clear from Fig. 1(c) that t_{opt} increases by N linearly. Finally, in the KR, J'_{opt} slowly decreases as N increases while it stays essentially constant in the DR [Fig. 1(d)]. This is commensurate with the expectation that $t_{opt} \propto e^{\alpha/\sqrt{J'_{opt}}}$ in the KR while, in the DR, t_{opt} and J'_{opt} are two independent quantities. The plot in Fig. 2(a) shows the exponential dependence of t_{opt} on $J'^{-1/2}_{opt}$ realized, in the KR, for long enough chains.

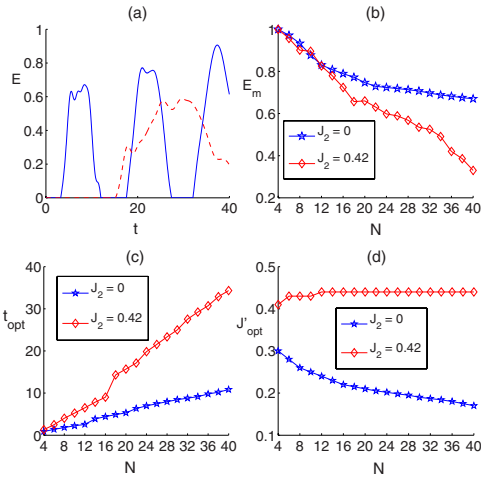

FIG. 1. (Color online) Comparing the Kondo ($J_2=0$) and the dimer ($J_2=0.42$) regimes. (a) Entanglement vs time for $N=30$ with $J'=0.19$ for the KR (solid line) and $J'=0.44$ for the DR (dashed line). (b) E_m vs the length N. (c) t_{opt} vs N. (d) J'_{opt} vs length N.

How the dynamics creates—even for very long chains of size N—high entanglement oscillations between the ending spins of the chain in the KR? To understand this, one should recall that, in the KR, the impurity spin forms an effective singlet with all the spins inside the cloud[9] and that, only in this regime, one can always choose J' so that ξ may be made

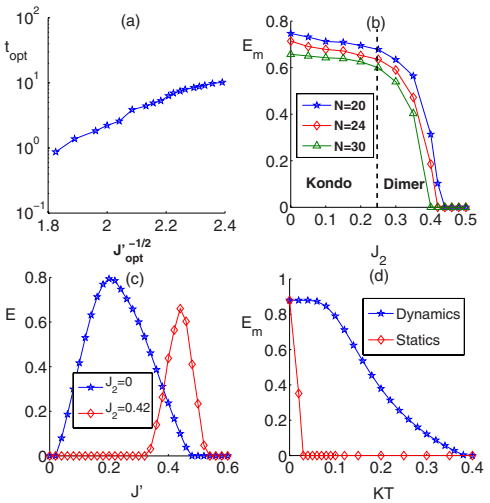

FIG. 2. (Color online) (a) t_{opt} vs $1/\sqrt{J'_{opt}}$ in the KR. (b) E_m at $t \propto 1/J'_{opt}$ vs J_2 for chains of different lengths. (c) Entanglement attained at t_{opt} vs J' for Kondo and DRs for $N=20$. (d) E_m vs temperature after bond quenching (blue line) and induced by (static) weak coupling with the rest of the chain (red line) for a chain of $N=10$.

comparable with N; at variance, in the DR, the impurity in $|GS_I\rangle$ picks out—no matter what the values of J' and N are—only an individual spin in the chain to form a singlet (a valence bond) while the remaining spins form singlets (*local dimers*) with their nearest neighbors.[7] Thus, only in the KR, one may use the remarkable resource to select—for any N—an initial state $|GS_I\rangle$ which is free from local excitations: in fact, when $\xi=N-2$ ($J'=J'_{opt}$) there is only a single entity, namely, the Nth spin, interacting with the impurity-cloud composite. As this situation can always be engineered by choosing, for any N, $J'=J'_{opt}$, this explains the distance-independent entanglement in Fig. 1(b). The proposed scenario provides an intuitive grasp on why, only in the KR, an optimal entanglement between the ending spins may emerge from quench dynamics as the result of the interplay between very few states. At variance, in the DR, the energy released by quenching is dispersed over the variety of different quantum modes associated to the local dimers.

To provide a more quantitative analysis, one may expand $|\psi(t)\rangle$ in terms of eigenvectors of H_F. By exact diagonalization (up to $N=20$), one finds that, in the KR, only two eigenstates of H_F (the ground state $|E_1\rangle$ and one excited state $|E_2\rangle$) predominantly contribute to the dynamics,

$$|E_1\rangle = \alpha_1|\psi^-\rangle_{1N}|\phi^-\rangle_b + \beta_1(|00\rangle_{1N}|\phi^{00}\rangle_b + |11\rangle_{1N}|\phi^{11}\rangle_b - |\psi^+\rangle|\phi^+\rangle_b),$$

$$|E_2\rangle = \alpha_2|\psi^-\rangle_{1N}|\phi^-\rangle_b - \beta_2(|00\rangle_{1N}|\phi^{00}\rangle_b + |11\rangle_{1N}|\phi^{11}\rangle_b - |\psi^+\rangle|\phi^+\rangle_b). \quad (3)$$

In Eq. (3), the first and the last spin are projected onto the singlet ($|\psi^-\rangle$) and the triplets ($|00\rangle$, $|11\rangle$, and $|\psi^+\rangle$) while the states of all spins in the body of the chain have been specified by the index b. After a time t, the state evolves as $|\psi(t)\rangle=\langle E_1|GS_I\rangle|E_1\rangle+e^{-i\Delta Et}\langle E_2|GS_I\rangle|E_2\rangle+\cdots$, where ΔE is the energy separation between the two levels. One defines $t=t_{opt}$ as the time for which the contribution of $|\psi^-\rangle_{1N}|\phi^-\rangle_b$ is most enhanced in $|\psi(t)\rangle$ due to a *constructive interference*.

The condition for the onset of constructive interference is

$$|\langle E_1|GS_I\rangle\beta_1| \approx |\langle E_2|GS_I\rangle\beta_2| \quad (4)$$

so that terms other than $|\psi^-\rangle_{1N}|\phi^-\rangle_b$ in $|\psi(t)\rangle$ do not contribute at $t=t_{opt}$. When $J'\to 1$ (very small cloud) then $|\langle E_1|GS_I\rangle|\approx 1$ while $|\langle E_2|GS_I\rangle|\approx 0$ as the ground state is hardly changed on quench: thus, constructive interference between $|E_1\rangle$ and $|E_2\rangle$ it is impossible. The condition of Eq. (4) cannot be satisfied also when $J'\to 0$ since one has now that $\beta_1\approx 0$ and $\beta_2\approx 1$ (the end spins form a singlet and triplet with each other in $|E_1\rangle$ and $|E_2\rangle$, respectively[12]). Thus, only for intermediate J' entanglement may peak. When $J'>J_{opt}(\xi<N-2)$—and particularly for $\xi<N/2$—the entanglement between the ending spins is frustrated by the existence of local excitations whose number increases as the size of the cloud gets smaller. In addition, when $J'<J_{opt}(\xi>N-2)$, the KC overtakes the chain and the Nth spin is already *included in the cloud* and its tendency is to screen the original impurity as in $|\psi^-\rangle_{1N}|\phi^-\rangle_b$ rather than to pair with it to form a spin one (as in the last three terms of $|E_1\rangle$). This makes β_1 quite small, and it becomes smaller as the cloud overtakes the chain and again the condition of Eq. (4) cannot be fulfilled. Consequently, the optimal situation is realized when $J'=J'_{opt}$ ($\xi=N-2$), i.e., just before the cloud overtakes the chain. Thus, only in the KR, one can convert—for any N—the *useless entanglement* between the impurity spin and the KC into a *usable entanglement* between the ending spins of the chain. The emerging long-distance entanglement analyzed in this Rapid Communication is, indeed, a genuine footprint of the presence of the KC in $|GS_I\rangle$ [Fig. 3(a)].

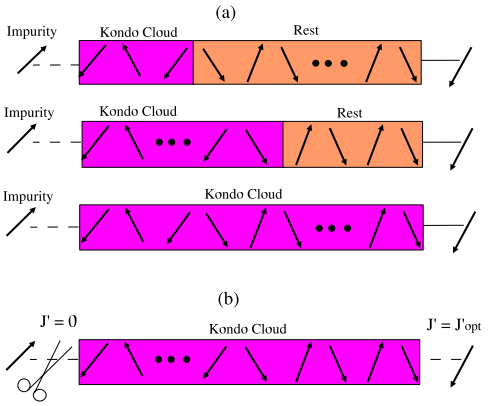

FIG. 3. (Color online) (a) Different $|GS_I\rangle$ for entanglement generation through quench dynamics. Top: the ground states with $\xi<N/2$ (no entanglement). Middle: $N/2<\xi<N-2$ (some entanglement). Bottom: $\xi=N-2$ (optimal entanglement). (b) Decoupling the impurity from the chain.

In Fig. 2(b), we plot—for both regimes—the entanglement reached after waiting for a time interval on the order of $1/J'_{opt}$. One notices that, for $J_2>J_2^c$, the entanglement peak decreases sensibly and goes to zero rather soon. The plot of the maximal entanglement vs J' is given in Fig. 2(c): here one sees that, in the KR, the entanglement rises from zero already at very small values of J'. This is expected since, in the KR, to a small J' is associated a large cloud containing the impurity sitting on the left side.

The essential role of the KC in the entanglement generation is further probed if one let evolve the ground state $|GS_I\rangle$ with a doubly quenched Hamiltonian obtained from Eq. (2) by isolating (i.e., putting $J'=0$) the left-hand side impurity while keeping fixed to J'_{opt} the bond connected to the last spin [see Fig. 3(b)]. This forbids the dynamical build up of that "portion" of the total entanglement which is only due to end to end effects. In Fig. 4, we have plotted the E_m vs N after double quenching in both regimes. Figure 4 shows that entanglement in the dimer phase collapses already when $N>12$ while it stays *unexpectedly* high—and almost independent on N—in the KR; the existing entanglement of the KC with the impurity[9] is dynamically swapped over to the last spin.

Note that a long-distance singlet between the end spins may be realized in a ground state when those spins are very weakly coupled ($J'=\epsilon/\sqrt{N}\ll 1/\sqrt{N}$) to a spin chain.[12] This static approach to generate entanglement relies on couplings which are so weak that they can merely be regarded as perturbations. Such entanglement is not robust against thermal fluctuations due to the smallness of the gap ($\propto J'^2=\epsilon^2/N$) between the ground state and a triplet state between the end

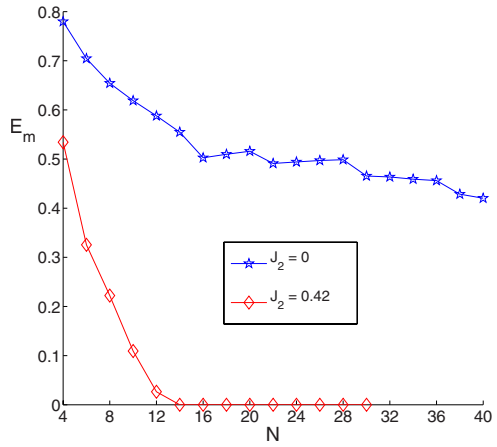

FIG. 4. (Color online) E_m vs N after decoupling the first impurity.

spins. On the other hand, our approach enables to generate entanglement dynamically even for J' as high as $J'_{opt} \approx 1/(\log N)^2$. Even when temperature is increased, the entanglement is not seriously disrupted till K_BT exceeds the Kondo temperature $[\propto 1/\xi = 1/(N-2)]$ after which the KC does not form. As a result, while in the dynamical approach $K_BT < 1/(N-2)$, in the static approach one has $K_BT < \epsilon^2/N$: thus, the long-distance entanglement generated through the dynamical approach is thermally more stable. For instance, for $\epsilon \sim 10^{-1}$, our dynamical approach is robust for temperatures 100 times higher than those required for the static approach. In Fig. 2(d), we plot E_m—as obtained in both approaches vs temperature for $N=10$. In the static approach, the ground state is replaced by the thermal state whereas in our dynamic approach, it is the initial state which is taken to be the relevant thermal state. We ignore thermalization and relaxation during dynamics since the dynamical time scale, set by t_{opt}, is fast enough (this is also an advantage over slow dynamical schemes with weak couplings[13]).

There are systems where the Kondo cloud mediated long-distance entanglement may be observed such as spin chains in ion traps,[20] with trapped electrons,[21] in chains of P donors in Si (Ref. 22) and Josephson chains with impurities.[23]

III. CONCLUSIONS

We have shown that substantial *long-range distance-independent* entanglement can be engineered by a nonperturbative quenching of a *single* bond in a Kondo spin chain. This is the first example where a *minimal local* action on a spin chain dynamically creates long-range entanglement. In contrast to all known schemes for entanglement between individual spins in spin chains, here the entanglement attains a constant value for long chains rather than decaying with distance. We showed that, in the KR, the entanglement between the ending spins is mediated by the KC. As the coupling is nonperturbative in strength $[J'_{opt} \approx 1/(\log N)^2]$, the entanglement generation is both fast and thermally robust. In the KR, the entanglement is periodic in time with a period of $2t_{opt}$—this is a curious instance of exciting a long-lived regular oscillation in a gapless system. Both, the long-distance entanglement mediated by the cloud and the periodic dynamics, provide a clear signature of the existence of the KC in a quantum system with impurities. These features are absent when the KC is absent, e.g., in the gapped DR. Beyond the spin-chain-based implementations, in principle, in double quantum dots, one may detect the length of the KC by inducing and probing the entanglement oscillations between the dots. Our analysis evidences that interesting applications to quantum information may arise exploiting relevant nonperturbative cooperative phenomena of condensed-matter physics.

Discussions with I. Affleck, A. Briggs, P. Calabrese, L. Campos Venuti, and M. B. Plenio are warmly acknowledged. A.B. and S.B. (P.S.) thank(s) the University of Perugia (UCL) for hospitality. A.B. and P.S. thank the GGI for hospitality and INFN for partial support. P.S. was supported by the ESF Network INSTANS. A.B. and S.B. are supported by the EPSRC, and S.B. is also supported by the QIPIRC (Grant No. GR/S82176/01), the Royal Society, and the Wolfson Foundation.

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